TRIGONOMETRY PROOFS

PURE MATHEMATICS

MIDLAND HIGH SCHOOL – MUKONO – MATHEMATICS DEPT

Mathematics department – 2025

Prove out the following

1. $\frac{cos\theta}{1-tan\theta}+\frac{sin\theta}{1-cot\theta}=sin\theta+cos\theta$
2. $tan2A=(sec2A+1)\sqrt{\sec^2 A-1}$
3. $sin3A+sin2A-sinA=4sinAcos\frac{A}{2}\cos\frac{3A}{2}$
4. $cotA=\frac{1}{2}\left(\cot\frac{A}{2}-\tan\frac{A}{2}\right)$
5. $\frac{1}{2}\left\{\frac{sin\theta}{1+cos\theta}+\frac{1+cos\theta}{sin\theta}\right\}=\frac{1}{sin\theta}$
6. $\frac{\tan^3 x}{1+\tan^2 x}+\frac{\cot^3 x}{1+\cot^2 x}=2cosec2x-sin2x$
7. $\frac{tanA}{1-cotA}+\frac{cotA}{1-tanA}=1+tanA+cotA=1+2cosecA$
8. $(1+\cos^2 2\theta)=2(\cos^4\theta+\sin^4\theta)$
9. $\frac{1+sinA-cosA}{1+sinA+cosA}=\sqrt{\frac{1-cosA}{1+cosA}}=\tan\frac{A}{2}$
10. $\frac{cosA}{1+sinA}+\frac{1+sinA}{cosA}=2secA$
11. $\cos(A+15^0)-\tan(A-15^0)=\frac{4cos2A}{1+2sin2A}$
12. $cosecA-2cot2AcosA=2sinA$
13. $\frac{\sin(n+1)A+2sin\,nA+\sin(n-1)A}{\cos(n-1)A-\cos(n+1)A}=\cot\frac{A}{2}$
14. $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=2tan2\theta$
15. $\frac{cosA-sinA+1}{cosA+sinA-1}=cosecA-cotA$
16. $\sin^8 x-\cos^8 x=\frac{1}{4}(3+cos4x)$
17. $\frac{sinA+cosA}{sinA-cosA}+\frac{sinA-cosA}{sinA+cosA}=-2cosec2A$
18. $\frac{sinA}{cotA+cosecA}=2+2\sin^2\frac{A}{2}=3-cosA$
19. $\frac{1+cos\theta+sin\theta}{1+cos\theta-sin\theta}=sec\theta+tan\theta$
20. $2(\sin^6\theta cos6\theta)-3(sin4\theta+cos4\theta)=-1$
21. $\cot^4\theta-1=cosec^4\theta-2cosec^2\theta$
22. $(1+cot\theta)(1+tan\theta+sec\theta)=2$
23. $\frac{1-cosA+cosB-\cos(A+B)}{1+cosA-cosB-\cos(A+B)}=\tan\frac{A}{2}\cot\frac{B}{2}$

24. $\frac{1+secA}{secA} = \frac{sin2A}{1-cosA}$

25. $\sin(10^0)\sin(30^0)\sin(50^0)\sin(70^0) = \frac{1}{16}$

26. $\sin^6\theta + cos6\theta + 3sin2\theta cos2\theta = 1$

27. $(\sin^8\theta - cos8\theta) = (sin2\theta - cos2\theta)(1 - 2sin2\theta cos2\theta)$

28. $\cot(A+B) = \frac{cotAcotB-1}{cotA+cotB}$

29. $\sqrt{\sec^2\theta + cosec^2\theta} = tan\theta + cot\theta = 2cosec\theta$

30. $\frac{\cos 2(x+\beta)+cos2x+cos2\beta+1}{cos2(x+\beta)-cos2x-cos2\beta+1} = cotxcot\beta$

31. $\frac{sin\theta sin\varphi}{cos\theta+cos\varphi} = \frac{2\tan\left(\frac{\theta}{2}\right)\tan\left(\frac{\varphi}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)\tan^2\left(\frac{\varphi}{2}\right)}$

32. $(cosecA - sinA)(secA - cosA) = \frac{1}{tanA+cotA}$

33. Show that; $\tan\frac{\pi}{8} = -1 + \sqrt{2}$

<u>Given the pre-condition</u>

34. If $cosec\theta - sin\theta = m$ and $sec\theta - cos\theta = n$, prove that;

    $(m^2n)^{\frac{2}{3}} + (n^2m)^{\frac{2}{3}} = 1$

35. The acute angles; x and y satisfy the following relationships;

    $2tanx = 1$ and $\sin(x+y) = \frac{7}{\sqrt{50}}$

    Show that the possible values of $tany$ are; **3** and $\mathbf{\frac{19}{3}}$

36. Given that; 2nd, 3rd and 4th terms of a Geometric series are; $cos\theta, \sqrt{2}sin\theta$ and $\sqrt{3}tan\theta$ respectively where $0^0 < \theta < \frac{\pi}{2}$.

    Show clearly, that the sum of the first 6 terms of the series is;

    $\frac{43}{12}(6+\sqrt{6})$

37. Given that; $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \frac{\pi}{2}$, show that;

    $xy + yz + zx = 1$

38. Given that; θ and β satisfy the following simultaneous equations;

    $\frac{sin2\theta}{1+sin\theta} = 1 - sin\beta$

    $\theta + \beta = \pi$

    Show that;

    a) $tan\theta = \frac{1}{2}$

    b) $\tan(3\theta + 5\beta) = -\frac{4}{3}$

39. Given that; $sin\theta = \frac{24}{25}$ and $cos\beta = \frac{15}{17}$. If θ is obtuse and β is reflex, show clearly that; $\sec(\theta + \beta) = \frac{425}{87}$
40. Given that; $4sin\theta + cos\theta = 2$ for $0^0 \leq \theta < 360^0$. Show that; $cos\theta = \frac{2 \pm 4\sqrt{13}}{17}$
41. If $\tan(x+y) = 2\tan(x-y)$, show that; $\frac{sin2x}{sin2y} = 3$
42. Show that; $\tan^{-1}\left[\frac{cos66^0 - sin48^0}{cos48^0 - sin66^0}\right] = -12^0$
43. Given that: $f(x) = \frac{1}{\sin^2 x + 3sinxcosx + 5\cos^2 x}$, show that; $\frac{2}{11} \leq f(x) \leq 2$
44. If $tan\theta + sin\theta = m$ and $tan\theta - sin\theta = n$, show that; $m^2 - n^2 = 4\sqrt{mn}$
45. If $acos\theta - bsin\theta = c$, prove that $(asin\theta + bcos\theta) = \pm\sqrt{a^2 + b^2 - c^2}$
46. If $cos\theta + sin\theta = \sqrt{2}cos\theta$, prove that: $cos\theta - sin\theta = \sqrt{2}sin\theta$
47. Given that; $8\sin(x-\beta) = 3\sin(x+\beta)$, prove that; $5tanx = 11tan\beta$
48. If $\left(\frac{x}{a}sin\theta - \frac{y}{b}cos\theta\right) = 1$ and $\left(\frac{x}{a}cos\theta + \frac{y}{b}\sin\theta\right) = 1$, prove that; $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 2$
49. If $(tan\theta + \sin\theta) = m$ and $(tan\theta - sin\theta) = n$, prove that; $(m^2 - n^2)^2 = 16mn$
50. If $cosec\theta - sin\theta = a^3$ and $sec\theta - cos\theta = b^3$, prove that; $a^2b^2(a^2 + b^2) = 1$
51. Given that: $\sin(x+y)\sin(x-y) = \frac{5}{36}$ and $cosx + cosy = \frac{5}{6}$. Show that; $\cos(x-y) = \frac{(1+\sqrt{6})}{6}$
52. If $a\cos^3\theta + 3a\sin^2\theta cos\theta = m$ and $a\sin^3\theta + 3asin\theta\cos^2\theta = n$, prove that; $(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}} = 2a^{\frac{2}{3}}$
53. If $sec\theta = x + \frac{1}{4x}$, prove that; $sec\theta + tan\theta = 2x\ or\ \frac{1}{2x}$
54. Given that; $\frac{1}{2}\sin^4 x + \frac{1}{3}\cos^4 x = \frac{1}{5}$. Show that; $\tan^2 x = \frac{2}{3}$
55. Given that; $7\cot^2 x + 6cotx = 1$ and $6tan\theta = 8 + \sec^2\theta$. Show that; $\tan(x+\theta) = -\frac{1}{2}$
56. Given that; $\frac{\sin(x-\beta)}{\cos(x-\beta) - 2tan\beta sin(x-\beta)} = tan\beta$, show that; $tanx = 2tan\beta$
57. Given that; $2cos\theta + sin\theta = 1$, show that the possible values of $7cos\theta + 6sin\theta$ are **2** and **6**
58. If $sin\theta + \sin^2\theta = 1$, prove that; $\cos^2\theta + \cos^4\theta = 1$
59. Prove that; $\frac{\cot(90^0 - A)}{tanA} + \frac{cosec(90^0 - A)sinA}{\tan(90^0 - A)} = \sec^2 A$

60. If $cosec\theta - cot\theta = p$, then show that; $\frac{p^2-1}{p^2+1} + cos\theta = 0$

61. Prove that; $\frac{sin3A}{1+2cos2A} = sinA$. Hence, show that; $sin15^0 = \frac{-1+\sqrt{3}}{2\sqrt{2}}$

62. If $x = psec\theta cos\beta, y = qsec\theta sin\beta$ and $z = rtan\theta$, then, show that-; $\frac{x^2}{p^2} + \frac{y^2}{q^2} - \frac{z^2}{r^2} = 1$

Inverse trigonometric functions

63. $\tan^{-1}(\sqrt{x}) = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

64. $\tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$

65. $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}\frac{3}{5}$

66. $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{8}{19}\right) = \frac{\pi}{4}$

67. Prove that; $\sin(2\sin^{-1}x + \cos^{-1}x) = \sqrt{(1-x^2)}$

68. Using **complex numbers**, prove that; $\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}(2) - \tan^{-1}(3) = \frac{\pi}{4}$

69. $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

70. $2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

71. $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

72. $\tan^{-1}\left(\frac{24}{7}\right) + 4\cot^{-1}(2) = \pi$

73. Given that; $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$, show that x has only **one** VALUE. Hence, state that value of x.

74. Prove that; $\tan^{-1}\left(\frac{m}{n}\right) - \tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}$

75. Prove that; $2tan^{-1}\left(\frac{1}{5}\right) + \cos^{-1}\left(\frac{7}{5\sqrt{2}}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

76. Prove that; $\tan^{-1}\left[\frac{\sqrt{1+sinx}-\sqrt{1-sinx}}{\sqrt{1+sinx}+\sqrt{1-sinx}}\right] = \frac{1}{2}x$

77. Prove that; $\tan^{-1}(8) + \tan^{-1}(2) + \tan^{-1}\left(\frac{2}{3}\right) = \pi$

78. Prove that; $\tan^{-1}\left[\sqrt{\frac{1-x}{1+x}}\right] = \frac{1}{2}\cos^{-1}x$

79. Prove that; $tan\left[\frac{1}{2}\sin^{-1}x\right] = \frac{1-\sqrt{1-x^2}}{x}$

80. If $A + B = 45^0$. Prove that; $(1+tanA)(1+tanB) = 2$. Hence, deduce that; $tan22\frac{1}{2}^0 = -1+\sqrt{2}$

Use of compound Angle formulae and factor formulae

In a triangle, ABC, prove that;

81. $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$
82. $2(cbcosA + cacosB + abcosC = a^2 + b^2 + c^2$
83. $\frac{sinB}{sinC} = \frac{c-acosB}{b-acosC}$
84. $cosA + cosB - cosC = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$
85. $tanA + tanB + tanC = tanAtanBtanC$
86. $sinB + sinC - sinA = 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
87. $\sin^3 A + sin^3 B + \sin^3 C = 3\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} + \cos\frac{3A}{2}\cos\frac{3B}{2}\cos\frac{3C}{2}$
88. $\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2} = \cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}$
89. $sinA + sinB + sinC = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
90. $sin2A + sin2B + sin2C = 4sinAsinBsinC$
91. $\cos^2 2A + \cos^2 2C + \cos^2 2B = 1 + 2cos2Acos2Bcos2C$
92. $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2} = 1 - 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
93. $cosA + cosB + cosC = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
94. $\tan(B-C) + \tan(C-A) + \tan(A-B) = \tan(B-C)\tan(C-A)\tan(A-B)$
95. $\frac{b^2-c^2}{a^2} = \frac{\sin(B-C)}{\sin(A+B)}$
96. $b + c = a\cos\frac{(B-C)}{2}cosec\frac{A}{2}$
97. $\frac{1}{a}\cos^2\frac{A}{2} + \frac{1}{b}\cos^2\frac{B}{2} + \frac{1}{C}\cos^2\frac{C}{2} = \frac{(a+b+c)^2}{4abc}$
98. $\frac{a+b-c}{a+b+c} = \tan\frac{A}{2}\tan\frac{B}{2}$
99. $\frac{bc}{ab+ac} = \frac{cosec(B+C)}{cosecB+cosecC}$
100. $3tanC = tanB$ if $\tan\frac{A}{2} = \left(1 + \tan^2\frac{A}{2}\right)\sin(B-C)$

END